Application of tolerance approach to fuzzy goal programming to aggregate production planning

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Abstract: This study presents the application of a tolerance approach to the fuzzy goal programming (FGP) developed by Kim and Whang (1998) and revised by Yaghoobi and Tamiz (2007-a) to aggregate production planning (RKW-APP) in a state-run enterprise of iron manufactures non-metallic and useful substances (Société des bentonites d'Algérie-BENTAL). The proposed formulation attempts to minimise total production and work force costs, inventory carrying costs and costs of changes in labour levels. A real-world industrial case study in demonstrating the applicability of the suggested model to practical APP decision problems is also given. The LINGO computer package has been used to solve the final crisp linear programming problem package and get an optimal production plan.

Keywords: aggregate production planning; fuzzy goal programming; tolerance approach.

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1 Introduction

Aggregate production planning (APP), sometimes called intermediate-range planning, involves production planning activities for six months to two years with monthly or quarterly updates. Changes in the workforce, additional machines, subcontracting, and overtime are typical decisions in APP.

The problem with APP concerns management's response to fluctuations in the demand pattern. Specifically, how can productivity, manpower and goods resources best be utilised in the face of changing demands to minimise the total cost of operations over a given planning horizon?

In response to changing demands, management can utilise the following strategies:

- adjust the work force through hiring and firing
- adjust the production rate through overtime and under-time absorb the demand fluctuation rate through inventory back logging or by allowing lost sales
- the production rate may be kept on a constant level and the fluctuations in demand met by altering the level of subcontracting.

Clearly, each of the above pure strategies implies a set of costs that may be both direct and opportunity. Changing the work force implies costs associated with hiring and layoff. Production rate changes entail costs of overtime and idle resource. Excess inventories require capital investment as well as direct costs, while shortages imply lost revenue and customer goodwill.

Any combination of these preceding strategies is, of course, also possible. The problem with APP is to select the strategy with least cost to the firm. This problem has been under extensive discussion, and several alternative methods for finding an optimal solution have been suggested in the literature.

2 Literature review

There are numerous methods available in the literature for APP. since Holt et al. (1955) proposed the HMMS rule in 1955, researchers have developed numerous models to help to solve the APP problem, each with its own pros and cons. According to Saad (1982), all traditional models of APP problems may be classified into six categories: (1) linear programming (LP) (Charnes and Cooper, 1961; Singhal and Adlakha, 1989); (2) linear decision rule (LDR) (Holt et al., 1955); (3) transportation method (Bowman, 1956); (4) management coefficient approach (Bowman, 1963), (5) search decision rule (SDR) (Taubert, 1968); and (6) simulation (Jones, 1967). When using any of the APP models, the goals and model inputs (resources and demand) are generally assumed to be deterministic/crisp, and only APP problems with the single objective of minimising cost over the planning period can be solved. The best APP balances the cost of building and taking inventory with the cost of the adjusting activity levels to meet fluctuating demand.

Masud and Hawang (1980) were the first to propose an APP model for the multiple product, single facility case where conflicting multiple objectives are treated explicitly. Three multiple decision-making methods are used to solve this problem, among them the Goal Programming (GP) model developed by Charnes and Cooper (1961).

In practice, the input data in the APP problem , as also data on demand, resources and cost as well as the objective function are frequently imprecise/fuzzy because some information is incomplete or unobtainable. Traditional mathematical programming techniques clearly cannot solve all fuzzy programming problems. Zimmerman (1976) first introduced the fuzzy set theory into conventional LP problems.

Many aspects of the APP problem and the solution procedures employed to solve APP problems lend themselves to the fuzzy set theory approach. Fuzzy APP allows the vagueness that exists in determining forecasted demand and the parameters associated with carrying charges, backorder costs, and lost sales to be included in the problem formulation. Fuzzy linguistic 'f-then' statements may be incorporated into the APP decision rules as a means for introducing the judgment and past experience of the decision maker into the problem. In this fashion, the fuzzy set theory increases the model's realism and enhances the implementation of APP models in the industry. The usefulness of the fuzzy set theory also extends to multiple objective APP models where additional imprecision due to conflicting goals may enter into the problem.

Wang and Fang (2001) present a novel fuzzy linear programming method for solving the APP problem with multiple objectives where the product price, unit cost to subcontract, work force level, production capacity and market demands are fuzzy in nature. An interactive solution procedure is developed to provide a compromise solution.

Reay-ChenWang and Tien-Fu Liang (2005) have developed a fuzzy multi-objective linear programming model for solving the multi-product APP decision problem in a fuzzy environment. Their formulation attempts to minimise total production costs, carrying and backordering costs and costs of changes in labour levels considering inventory level, labour levels, capacity, warehouse space and the time value of money.

Abouzar Jamalnia and Mohammad Ali Soukhakian (2009) have developed a hybrid (including qualitative and quantitative objectives) fuzzy multi-objective non-linear programming model with different goal priorities for solving an APP problem in a fuzzy environment. The proposed model tries to minimise total production costs, carrying and back ordering costs and costs of changes in the workforce level (quantitative objectives) and maximise total customer satisfaction (qualitative objective) with regard to the inventory level, demand, labour level, machine capacity and warehouse space their formulation based on FGP developed bay Chen and Tsai (2001).

This study presents an application of the APP-based A tolerance approach to the fuzzy goal programming (FGP) developed by Kim and Whang (1998) and Revised by Yaghoobi and Tamiz (2007-a) and its application in the national firm of iron manufactures non- metallic and useful substances for solving the problems of the APp. The proposed model minimises total production and work force costs, cost of inventory and minimises the degree of change in the work force.

3 Basic structure of the GP model

3.1 Definition and literature of GP

The initial development of the concept of GP was due to Charnes and Cooper, in a discussion which took place in 1961, although they claim that the idea actually originated in 1952. In essence, they proposed a model and approach for dealing with certain linear programming problems in which conflicting "goals of management were included as constraints (Ignizio, 1976; Romero, 1991).

The essential activity of a manager is decision-making. This activity is becoming more complex because managers (decision-makers) try to integrate into their own decisions many different factors. Multiple-criteria problems in conferences, in academic publications and in practice have increased in importance (Martel and Aouni, 1990). GP can be considered to be a mathematical programming method and a member of the multi-criteria decision-making MCDM family. GP constitutes of a modification and extension of linear programming. These two programming techniques are similar to the fact that they both represent optimal solutions to goals and constraints. Nevertheless, GP and linear programming have significant performance differences that give the advantage to GP, which is due to the greater scale of problems that is applied (Zeleny, 1981, 1982).

GP is a multi-objective programming (MOP) technique. GP is based on the distance function concept (Romero, 1991). It later became the most popular model of the MOP. Its popularity is due to the fact that it is a simple model, easy to apply, and takes advantage of the extensive number of mathematical programming software available in the market (Aouni and Kettani, 2001).

Until the middle of the 1970s, GP applications reported in the literature were rather scarce. Since that time, and chiefly due to seminal works by Lee (1972) and Ignizio (1976), an impressive boom in GP applications and technical improvements has arisen. It can be said that GP has been, and still is, the most widely used multi-criteria decision-making technique (Romero, 1991). Although Schniederjans (1995) has detected a decline in the life cycle of GP with regard to theoretical developments, the number of cases along with the range of fields to which GP has been, and is still is, applied is impressive, as shown by recent surveys by Romero (1986), Schniederjans (1995) and Tamiz et al. (1993). It has been applied successfully in practice for many years (Jones and Tamiz, 2002). GP models aim to minimise deviations of the objective values from aspiration levels specified by decision maker(s) (Yaghoobi and Tamiz, 2007-b). The variants of GP are numerous, and contain many different sub-areas which can bewilder practitioners with no knowledge of GP, but wish to apply it to their multi-objective real world situation (Tamiz et al., 1998).

3.2 Formulation of the GP model

Before we can define the GP model, it is absolutely essential to establish precise definitions for certain keywords and concepts. This is particularly critical where such definitions differ or must be made sharper than in conventional mathematical programming. Now, since the definitions of such terms as variables (i.e., controllable/noncontrollable; continuous/discrete), functions (i.e., linear and nonlinear); equations inequalities; and mathematical models are the same as in the multi-objective area, we may move directly to the following set of definitions (Ignizio, 1983).

– ***Objectives*:** objectives are represented by mathematical functions of their decision or control variables. Such functions usually represent some desire or wish of the decision maker(s). It is important to note that the value of an objective function is left unspecified. The two most common objective function forms are: maximise $f(x)$ or minimise $f(x)$.

– ***Aspiration level*:** an aspiration level is a specific (realistic) value (or 'target' level) associated with a desired or acceptable level of achievement of an objective. Thus, an aspiration level may be used to measure the achievement or non-achievement of an objective.

– ***Goal*****:** an objective in conjunction with an aspiration level is termed a goal. That is, if we say that we wish to maximise profit, then that is an objective.

However, if we instead, wish to achieve a profit level of at least $1000, we have established a goal. The mathematical form of a goal is either:

$$
\begin{aligned}
& \textit{satisfy} \quad f(x) \le b \\
\textit{or}, \quad & \textit{satisfy} \quad f(x) \ge b \\
\textit{or}, \quad & \textit{satisfy} \quad f(x) = b
\end{aligned}
$$

depending on the situation.

– ***Constraint*****:** a constraint has exactly the same mathematical appearance as a goal. However, in multi-objective mathematical programming, a constraint is a subset of the concept of goals. In specific, a constraint is an inflexible (or rigid or hard) goal. Thus, when a truly inflexible constraint is encountered, we shall denote this relationship as a rigid constraint or, alternately, as an inflexible or absolute goal.

In conventional (i.e., single objective) mathematical programming, we did not have to worry about the distinctions between objectives and goals, or between goals and rigid constraints, as there we dealt with only objectives and (rigid) constraints. However, in multi-objective mathematical programming, precise, non-ambiguous definitions are necessary and, in fact, help to form the basis of the power and flexibility of many of the multi-objective methods.

As we have noted in the previous section, generalised GP encompasses any method which converts the baseline model of into a model consisting solely of goals (some flexible and some rigid). This is the single, distinguishing feature of generalised GP. The distinction between various types of generalised GP is made on the basis of how one actually measures the 'goodness' of any solution (value of b) to the set of goals. This is typically facilitated by means of the concepts of 'goal deviations' and the 'achievement function'.

– ***Goal deviations*****:** There are, as discussed, three forms of goals: $f(x) \le b$, $f(x) \ge b$, *and* $f(x) = b$. Since we are using the philosophy of 'satisficing', we are only interested (at least initially) in measuring the non-achievement of each goal. This is the unwanted deviations from the aspiration levels (i.e., the value of each 'b'). We let d be the deviation from the goal aspiration and, since such deviation may be either a negative or a positive value d, we let: $d = n + p$, where $n^* p = 0$ and $n, p \ge 0$.

Typically, n_i is known as the negative deviation of goal i, while p_i, is the positive deviation. Thus, to satisfy a specific goal, we attempt to minimise the unwanted component (or components) of the goal deviation. This is summarised in Table 1, below:

Table 1 Goals and coal deviations

Initial form of goal	*Converted form*	*Deviation variables to be minimised*
$f(x) \le b$	$f(x) + n - p = b$	p
$f(x) \ge b$	$f(x) + n - p = b$	n
$f(x) = b$	$f(x) + n - p = b$	$n + p$

Charnes and Cooper (1961) illustrated how that deviation could be minimised by placing the variables representing deviation directly in the objective function of the model. This allows multiple goals to be expressed in a model that will permit a solution to be found. Multiple and conflicting goals are a distinguishing characteristic to describe how a GP model differs from a linear programming model.

– ***Model*****:** Charnes and Cooper (1978) presented a generally accepted statement of a GP model, as follows:

$$\begin{aligned}
&Min\,Z = \sum_{i=1}^{k}(n_i + p_i)\\
&Subject\,to:\\
&f_i(x_j) + n_i - p_i = b_i \qquad for \quad i = 1,.............,k \qquad (1)\\
&C_x \le c \qquad (system \;\; constraints)\\
&n_i, p_i, x_j \ge 0, \qquad for \quad i = 1,....,k \quad j = 1,.......,m
\end{aligned}$$

n_i: is called a positive deviation variable or over-achievement of goal b_i.
p_i: is called a positive deviation variable or over-achievement of goal b_i.
b_i: is the arithmetic value of goal i.
Z: is the sum of all deviations. The deviation variables are related to the functionals where:

$$n_i = \frac{1}{2}\left[\left|b_i - f_i(x_j)\right| + (b_i - f_i(x_j))\right]$$

$$p_i = \frac{1}{2}\left[\left|b_i - f_i(x_j)\right| - (b_i - f_i(x_j))\right]$$

Then the sum of the deviations gives:

$$\begin{aligned}
n_i + p_i &= \frac{1}{2}\left[\left|b_i - f_i(x_j)\right| + (b_i - f_i(x_j))\right] + \frac{1}{2}\left[\left|b_i - f_i(x_j)\right| - (b_i - f_i(x_j))\right]\\
&= d = \left|b_i - f_i(x_j)\right|
\end{aligned}$$

4 Basic structure of Fuzzy Goal Programming (FGP)

4.1 Definition and literature review of FGP

GP models have been classified based on the achievement function that is used to combine unwanted deviations (Romero, 2004): (1) weighted GP (also known as 'non-pre-emptive GP') where the weighted sum of deviations from the targets are minimised; (2) pre-emptive priority GP (also known as 'lexicographic GP'), where a deviation from a higher priority level goal is considered to be infinitely more important than a deviation from a lower priority goal, and (3) MINMAX GP (also known as 'Chebyshev GP'), where minimisation of the maximum weighted deviation from the target values is sought. However, determining precise aspiration levels for the objectives in real world problems often is a difficult task for decision maker(s) (Yaghoobi and Tamiz, 2007-b). In fact, there are many decision-making situations where the DM does not have complete information on some parameters and, in particular, the goal values in the GP model (Aouni et al., 2010).

The literature review reveals that the FGP is one of the GP variants. According to this review, we notice that the majority of the FGP formulations and applications are based on the model developed by Hannan (1981-a, 1981-b).

Bellman and Zadeh (1970) set the basic principles of decision making in fuzzy environments, which have been used as building blocks of fuzzy linear programming. The use of membership functions in the GP based on the fuzzy set theory was first carried out by Zimmerman (1976, 1978, 1983) and Narasimhan (1980). Further extensions were provided by Hannan (1981-a, 1981-b), Ignizio (1982-a), and Tiwari et al. (1987). Since the early 1980s, fuzzy sets have been used in GP models to represent the satisfaction degree of the decision maker with respect to his/ her preference structure (Narasimhan, 1980); Hannan, 1981-a; Tiwari et al., 1987; Mohamed, R.H 1997;, Chen and Tsai (2001); Yaghoobi and Tamiz (2007-b); and to represent uncertain knowledge about a certain parameter (Mohandas et al., 1990, Chanas and Kuchta (2002).

Various approaches to treating the relative importance of goals in FGP models have been developed. Narasimhan (1980) used a combination of linguistically defined weights, such as 'very important', 'moderately important' and achievement degrees of the goals. The weights and achievement degrees are combined by defining a membership function for each linguistic weight, where desirable achievement degrees are specified to represent goal importance. Hannan (1981-b) showed that the above composite approach may lead to some contradictory results, and suggested the use of explicitly defined weights to represent the relative importance of goals. Hannan (1981-a) proposed a fuzzy logic-based methodology that employs piecewise linear functions, which represent the decision–maker's satisfaction with attainment of goal values. A target achievement degree is determined for each goal and the problem is converted to a standard GP formulation, where deviations from these target values are minimised using standard pre-emptive, weighted or MINMAX achievement functions. A different approach is proposed by Tiwari et al. (1987). The authors considered an additive FGP model with relative importance of commensurable goals.

The model included a single-objective function defined as the weighted sum of achievement degrees of the goals with respect to their target values. Based on piecewise linear approximation (PLA), Yang et al. (1991) have further formulated the problem with fewer variables, which can yield the same solutions as Narasimhan's and Hannan's model. Kim and Whang (1998) have proposed an FGP formulation where the concept of tolerances is introduced to express the fuzzy goals of the DM, instead of using the conventional membership functions. Chen and Tsai (2001) proposed an extension of the additive model to consider goals of different importance and pre-emptive priorities, where the relative importance of goals is modelled by corresponding desirable achievement degrees. Recently, Yaghoobi and Tamiz (2007-b) have proposed a more efficient formulation, and they have highlighted the fact that the model of Kim and Whang (1998) is different from the Hannan (1981-a, 1981-b) model. It is proved that the proposed model is an extension to the Hannan model that deals with unbalanced triangular linear membership functions. In addition, it is shown that the new model is equivalent to a model proposed by Yang et al. (1991).

Until the middle of the 1990s, FGP applications reported in the literature were rather scarce. We list a categorisation of the major applications of the FGP within management and economics below: Curve and response surface fitting, Media planning, Manpower planning, Programme selection, Project selection, Hospital administration, Academic resource allocation, Municipal economic planning, Transportation problems, Energy/water resources, Radar system design, Sonar system design, Planning in wood products, Portfolio selection, Determination of time standards, Development of cost estimating relationships, Urban renewal planning, Merger strategies, Multi-plant/product aggregate production loading, BMD systems design, Multi-objective facility location, Free flight rockets, Solar heating and cooling, Natural gas well siting and Maintenance level determination. All of these applications have one thing in common: they could be forced into a traditional single-objective model if one so wished. However, those investigating these problems believed that they truly involved multiple, conflicting objectives, and thus, were most naturally modelled as a FGP problem.

4.2 Formulation of FGP

A useful tool for dealing with imprecision is the fuzzy set theory (Zadeh 1965). An objective with an imprecise aspiration level can be treated as a fuzzy goal. Initially, Narasimhan incorporated the fuzzy set theory in GP in 1980 and presented an FGP model (Narasimhan 1980). Hannan simplified Narasimhan's method to an equivalent simple linear programming in 1981 (Hannan 1981-b). These pioneering works led to extensive research in the use and application of FGP to real life problems.

To solve FGP problems, various models based on different approaches have been proposed. A survey and classification of FGP models has been presented by Chanas and Kuchta (2002). There are three types of fuzzy goals that are the most common. The following FGP model contains these fuzzy goals.

$$\begin{aligned} OPT \ \ (AX)_i &\underset{\approx}{\leq} b_i \quad i = 1,............, i_O \\ (AX)_i &\underset{\approx}{\geq} b_i \quad i = i_O + 1,......, j_0 \\ (AX)_i &\cong \quad i = j_O + 1,......, K \\ X &\in C_{S,} \end{aligned} \tag{2}$$

where OPT means finding an optimal decision X such that all fuzzy goals are satisfied, $AX)_i = \sum_{j=1}^{n} a_{ij} x_j ....i = 1,......., k$, b_i is the aspiration level for *i.th* goal and the symbol $\cong$ is a fuzzifier representing the imprecise fashion in which the goals are stated.

The integrated use of GP and the fuzzy sets theory has already been reported in the literature. Zimmerman (1976), Hannan (1981-a; 1981-b), Leberling (1981), Rubin and Narasimhan (1984), Tiwari et al. (1987), Wang and Fu (1997), Kim and Whang (1998), Chen and Tsai (2001), Yaghoobi and Tamiz (2007-b), Yaghoobi et al. (2009), Jiminez et al. (2007), Hatami and Tavana (2011) further integrated several fuzzy linear and multi-objective programming techniques.

The approach chosen in this study for application to the problem of APP is similar to the method developed by Kim and Whang (1998) and revised by Yaghoobi and Tamiz (2007-a).

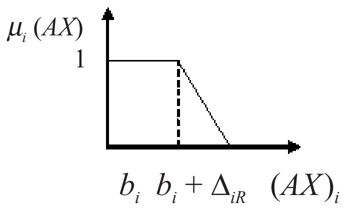
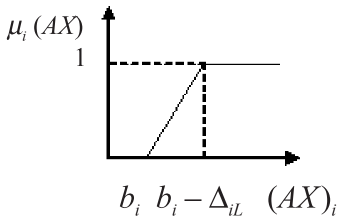
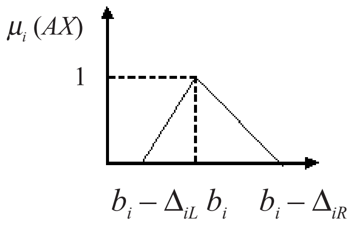
4.3 Membership function

The concept of membership functions, based on the fuzzy set theory, has been introduced and used by Zimmerman (1976; 1978; 1983) and Freeling (1980) for modelling fuzziness related to decision-making context parameters. The general formulation of the membership function used in their formulation is defined and depicted as follows (Figure 1, type 1 and type 2).

Narasimhan (1980) and Hannan (1981-a) were the first to give a FGP formulation by using the concept of the membership function. This function is defined on the interval [0, 1]. Thus, the membership function for the $i-th$ goal has a value of 1 when this goal is attained, and the decision maker is totally satisfied; otherwise, the membership functions assume a value between 0 and 1.

Linear membership functions are used in theory and practice more than other types of membership functions. For the above three types of fuzzy goals, linear membership functions are defined and depicted as follows (Figure 1):

Figure 1 Linear membership function and analytical definition

Membership function	**Analytical definition**
	$\mu_i\;(AX)_i=\begin{cases}1 \ldots\ldots \text{if}\;(AX)_i\le b_i\\ \dfrac{(AX)_i-b_i}{\Delta_{iR}} \ldots \text{if}\;b_i\le(AX)_i\le b_i+\Delta_{iR} \ldots i=1,\ldots,i_0 \ldots(1)\\ 0 \ldots\ldots \text{if}\;(AX)_i\ge b_i+\Delta_{iR}\end{cases}$
Type 1	
	$\mu_i\;(AX)_i=\begin{cases}1 \ldots\ldots \text{if}\;(AX)_i\ge b_i\\ \dfrac{b_i-(AX)_i}{\Delta_{iL}} \ldots \text{if}\;b_i\le(AX)_i\le b_i+\Delta_{iL} \ldots i=i_0+1,\ldots,j_0 \ldots(2)\\ 0 \ldots\ldots \text{if}\;(AX)_i\le b_i+\Delta_{iL}\end{cases}$
Type 2	
	$\mu_i\;(AX)_i=\begin{cases}0 \ldots\ldots \text{if}\;(AX)_i\le b_i\\ \dfrac{(AX)_i-b_i}{\Delta_{iR}} \ldots \text{if}\;b_i\le(AX)_i\le b_i+\Delta_{iR}.\quad i=j_0+1,\ldots,k_0\\ \dfrac{b_i-(AX)_i}{\Delta_{iL}} \ldots \text{if}\;b_i\le(AX)_i\le b_i+\Delta_{iL}\\ 0 \ldots\ldots \text{if}\;(AX)_i\ge b_i+\Delta_{iR}\end{cases} \ldots(3)$
Type 3	

where Δ_{iR} (or Δ_{iL}) is the quantity of a tolerance in the case of fuzzy goal. This quantity is specified by the decision makers (DMs).

4.4 MINMAX approach to FGP problems

In conventional GP problems, the aspiration (target) levels are determined precisely. GP models attempt to minimise the deviations from precise aspiration levels to find an optimal solution for GP problems. Consider a GP problem that is the same as FGP problems, but without the symbol ≈. There exist two major models in GP which are most widely used:

- Weighted GP (WGP)
- MINMAX GP

MINMAX GP was introduced by Flavell in 1976. This approach minimises the maximum deviation from any single goal. It provides an optimal solution that represents the most balanced solution among the achievements of different goals. Hannan(1981-a) introduced the first MINMAX approach to FGP based on the MINMAX GP developed by Flavel (1976). He considered all fuzzy goals of type 3, Figure 1, with isosceles triangular membership functions ($\Delta_{iR} = \Delta_{iL} = \Delta_i$). The linear programming for this special case of FGP problems is as follows:

$$\begin{aligned} &Max \quad Z = \mu \\ &subject\ to: \\ &f_i(x)/\Delta_i + \delta_i^- - \delta_i^+ = g_i/\Delta_i \quad (for\ i = 1, 2, ...p) \\ &\mu + \delta_i^- + \delta_i^+ \leq 1 \qquad (for\ i = 1, 2, ...p) \\ &x \in X \\ &\mu, \delta_i^- and\ \ \delta_i^+ \geq 0 \end{aligned} \tag{3}$$

where μ is the degree of membership function.

Despite the fact that the FGP model allows imprecision modelling related to goals values, this model seems to be rigid. Ignizio (1982-b) stresses the fact that Narasimhan and Hannan's formulations are limited to specific cases where the decision maker (DM) is supposed to have membership functions of particular forms like the triangular one. The use of such triangular membership functions was mainly criticised by Ignizio (1982-b) and Martel and Aouni (1998). These criticisms are related to the fact that the triangular form of membership functions does not adequately reflect the DM's preferences, and are not an appropriate way for modelling the goal's fuzziness. Wang and Fu (1997), Pal and Moitra (2003) and Chen and Tsai (2001) have some concerns regarding the way to deal with goals fuzziness through the triangular form of the membership functions, and indicate that in some applications, this type of function leads to non-desirable results.

4.5 Weighted additive FGP (WAFGP)

Hannan (1981-a, 1981-b) introduced the first weighted FGP. He considered all fuzzy goals of type 3, Figure 1, with isosceles triangular membership functions ($\Delta_{iR} = \Delta_{iL} = \Delta_i$). The linear programming for this special case of FGP problems is as follows:

$$\begin{aligned}
&Min \quad Z = \sum_{i=1}^{P} (w_i^- n_i + w_i^+ p_i) \\
&subject\ to: \\
&f_i(x)/\Delta_i + n_i - p_i = b_i/\Delta_i \quad (for\ i = 1,2,.......p) \\
&\lambda + n_i + p_i \le 1 \qquad (for\ i = 1,2,.......p) \\
&x \in X \\
&\lambda, n_i \ and \ \ p \ge 0
\end{aligned} \tag{4}$$

where w_i^+ and w_i^- are the relative importance coefficients associated with the positive and the negative deviations, respectively. These weights reflect, partially, the importance that the decision maker (DM) can express differently, depending on whether there is an over- or an under-achievement of the objective.

Tiwari et al. (1987) proposed an alternative formulation based on an WAFGP. They use the addition as an operator to aggregate the weighted membership function values. In their model, only fuzzy goals of types (1-2) are considered. Their model with the notations used in this paper is as follows:

$$\begin{aligned}
&Max\ Z = \sum_{i=1}^{K} w_i \mu_i \\
&st. \\
&\mu_i = 1 - \frac{(AX)_i - b_i}{\Delta_{iR}} \qquad i = 1,..........,i_0 \\
&\mu_i = 1 - \frac{b_i - (AX)_i}{\Delta_{iL}} \qquad i = i_0 + 1,..........,K \\
&0 \le \mu_i \le 1 \qquad\qquad i = 1,............,K \\
&X \in C_S
\end{aligned} \tag{5}$$

In Yaghoobi and Tamiz (2006), it is proved that model (5) sometimes yields suboptimal solutions and model (6) overcomes this weakness. Another advantage of model (6) is that in the optimal solution μ_i determines the degree of membership function for the i th fuzzy goal.

$$\begin{aligned}
&Max\ Z = \sum_{i=1}^{K} w_i \mu_i \\
&st. \\
&\mu_i \le 1 - \frac{(AX)_i - b_i}{\Delta_{iR}} \qquad i = 1,..........,i_0 \\
&\mu_i \le 1 - \frac{b_i - (AX)_i}{\Delta_{iL}} \qquad i = i_0 + 1,..........,K \\
&0 \le \mu_i \le 1 \qquad\qquad i = 1,............,K \\
&X \in C_S
\end{aligned} \tag{6}$$

Among the most important criticism of previous models is that they do not use all types of membership functions, but only use type 1 and type 2, which makes them of limited use in some applications. They do not incorporate the DM's preferences.

4.6 A tolerance approach to FGP (RKW model)

Kim and Whang (1998) have proposed another approach based on the weighted additive model for solving FGP problems with unequal weights, which can be formulated as a single LP problem with the concept of tolerances. They attempted to extend the Hannan (1981-b) model by introducing an LP model that is able to handle unbalanced triangular linear membership functions. The Kim and Whang (KW) model for FGP can be formulated as follows:

$$
\begin{aligned}
&Min \quad Z=\sum_{i=1}^{io} w_i\beta_i^+ + \sum_{i=i_0+1}^{j_0} w_i\beta_i^- + \sum_{i=j_0+1}^{k} w_i(\beta_i^+ + \beta_i^-)\\
&st:\\
&(AX)_i - \Delta_{iR}\,\beta_i^+ \le b_i \qquad i=1,\dots,i_0\\
&(AX)_i + \Delta_{iL}\,\beta_i^- \ge b_i \qquad i=i_0+1,\dots,j_0\\
&(AX)_i + \Delta_{iL}\,\beta_i^- - \Delta_{iR}\beta_i^+ = b_i \qquad i=j_0+1,\dots,K\\
&\beta_i^+,\beta_i^- \ge 0 \qquad i=1,\dots.,K\\
&X\in C_S\ ;
\end{aligned}
\tag{7}
$$

where w_{i} is the weight of the *ith* goal and β_i^+ and β_i^- are the positive and negative deviational variables.

However, Yaghoobi and Tamiz (2007-a), in a recent note, have shown that this model can yield undesirable results in comparison with the Hannan (1981-b) model. It is suggested to insert the following constraints into the model:

$$
\begin{aligned}
&\beta_i^+ \le 1..........i=1,........,i_0\\
&\beta_i^- \le 1..........i=i_0+1,......,j_0\\
&\beta_i^+ + \beta_i^- \le 1......i=j_0+1,.....,K
\end{aligned}
\tag{8}
$$

Model (7) augmented with (8) is called the revised Kim and Whang (RKW) model.

- The operator (min sum of tolerances: max sum of all goals membership degrees) of the RKW model is more suitable to unbalanced development planning than the max–min operator of other FGP models. That is because in solving a FGP problem, the feasible solution region approached by the RKW model is larger than or equal to those of the other FGP models like Narasimhan (1980), Hannan (1981-b), Yang et al. (1991) and Tiwari et al. (1987). If we solved a given FGP problem, the sum of the membership degrees of the optimal solution achieved by the RKW model would be better than or equal to those of the other FGP models.

- In addition, when comparing degree differences between the grade of membership for the best satisfied goal and the grade for the worst satisfied goal, the difference solved by the RKW model is greater than or equal to those by other FGP models.
- The RKW model can be used for types 1–3 of membership functions.

5 Multi-objective programming (MOP) model to APP

5.1 Parameters and constants definition

v_{it}: production cost for product *i* in period *t*, excluding labour cost in period *t* (units)
c_{it}: inventory carrying cost for product *i* between period *t* and *t* + 1
r_t: regular time work force cost per employee hour in period *t*
d_{it} : forecast demand for product *i* in period *t* (units)
K_{it}: quantity to produce one worker in regular time for product *i* in period *t*
I_{io}: initial inventory level for product *i* (units)
T: horizon of planning
N: total number of products P_{it}: quantity of *i* product to the period *t*
I_{it}: inventory level for product *i* in period *t* (units)
H_t: worker hired in period *t* (man)
F_t: workers laid off in period *t* (man)
$I_{it.Min}$: minimum inventory level available for product *i* in period *t* (units)
W_t: total strength of work force level in period *t* (man)
W_{Min}: the minimum work force level (man) available in period *t*
W_{Max}: the maximum work force level (man) available in period *t*

5.2 Objective functions

Masud and Hwang (1980) specified three objective functions to minimise total production costs, carrying and backordering costs and costs of changes in labour levels. In this study, we propose a model that will be using two strategies, where they are available, in a national firm dealing in iron manufactures and non-metallic and useful substances. In their multi-product APP decision model, the three objectives of the APP model can be formulated as follows:

- **Minimise total production costs**

$$Min.Z_1 \cong \sum_{i=1}^{N}\sum_{t=1}^{T}(v_{it}P_{it}) + \sum_{t=1}^{T}(r_t W_t)$$

The production costs include: regular time production, overtime, carrying inventory, and specify the costs of change in work force levels.

- **Minimise costs of changes in labour levels**

$$Min.Z_2 \cong \sum_{t=1}^{T} h_t H_t + f_t F$$

- **Minimise carrying costs**

$$Min.Z_3 \cong \sum_{t=1}^{T}(c_{it} I_{it})$$

.

where the symbol ≅ is the fuzzified version of = and refers to the fuzzification of the aspiration levels.

The objective functions of the APP model in this study assume that the DM has such imprecise goals as, the objective functions should be essentially equal to some value. These conflicting goals are required to be simultaneously optimised by the DM in the framework of fuzzy aspiration levels.

5.3 Constraints

- **The inventory level constraints:**

  $P_{it} + I_{i,t-1} - I_t = d_t$

  $I_{it} \geq I_{it.\mathrm{Min}}$

- **Constraints on labour levels:**

  $W_t - W_{i-1} - H_i + F_t = 0$

  $W_{\mathrm{Min}} \leq W_t \leq \mathrm{W}_{\mathrm{Max}}$

- **Constraints on labour capacity in regular and overtime:**

  $P_{it} - K_t * W_t \leq 0$

- **Non-negativity constraints on decision variables:**

  $P_{it}, I_t, W_t, H_t, F_t \geq 0$

6 RKW model for APP (RKW-APP)

We will use the method that was developed by Kim and Wahang (1998) (models 7, 8) and revised by Yahgoobi and Tamiz (2007-a) (constraint 6) for formulating the APP problem with fuzzy gaols. The complete RKW-APP model can be formulated as follows.

$$Min \quad Z_4 = \sum_{i=1}^{3} w_i \beta_i^+$$

$$ST:$$

$$Z_1 - \Delta_{IR}\beta_1^+ \leq b_1 \quad (Minimize\ total\ production\ \cos ts)$$

$$Z_2 - \Delta_{IR}\beta_2^+ \leq b_2 \quad (Mininmize\ \cos tes\ of\ changes\ in\ labor\ leveles)$$

$$Z_3 - \Delta_{IR}\beta_3^+ \leq b_3 \quad (Minimize\ carying\ \cos ts)$$

$$X_{it} + I_{i,t-1} - I_{it} = d_{it}$$

$$I_{it} \geq I_{it.Min}$$

$$W_t - W_{t-1} - H_t + F_t = 0$$

$$W_{Min} \leq W_t \leq W_{Max}$$

$P_{it} - K_{it} * W_t \leq 0$

$\beta_i^+ \leq 1$

$P_{it}, I_{it}, W_t, H_t, \beta_i^+ \geq 0$

7 Model implementation

7.1 An industrial case study and data description

In this section, as a real-world industrial case, we use a data set provided by the national firm dealing in iron manufactures, non- metallic and useful substances (BENTAL) in Algeria. This company manufactures three types of products which are important, and one of the raw materials used in many industries, with bentonite (BEN), carbonate of calcium (CAL) and discolouring (TD). The firm employs 175 workers, and the system of work in the firm is continuous production (8 × 3 hours) for all days of the week except Thursday, a half-working day and Friday, which is a rest day. Production management is composed of 68 workers divided into 3 groups.

The demand for the products of the individual firm in the production of mineral products mentioned above is large, which may cause problems in the productive capacity of this firm. Figure 2 show fluctuations in demand on the level of monthly production capacity of any production capacity (CAP).

Therefore, the impact on the level and volatility of productive capacity calls for the firm, in an attempt to develop a plan of production, to try to cope with fluctuations in demand due to seasonal changes. Table 2 summarises the basic data gathered from the firm. The proposed model implementation in the company has the following conditions:

1. There is a six-month period planning horizon.
2. A three product situation is considered.
3. The initial inventory in period 1 is I_{10} = 1857 tons of BEN, I_{20} = 1029 tons of TD and I_{30} = 1860 tons of CAL.
4. Minimum inventory that must be maintained during the period *t* of product *i* is 500. *Tons*.
5. The costs associated with hiring and layoff, according to estimations of the human resource management department, are respectively 51,780 DA/man and 41,550 DA/man.
6. The cost of one worker in the production of three products during the *t* period is r_t = 26940.706.*DA*/*man*.
7. The minimum work force level (man) available in each period is W_{Min} = 55 workers.
8. The maximum work force level available in each period is W_{Max} = 68 workers.
9. The initial worker level is (W_0 = 56).
10. The maximum capacity of storage of the 3 products in the firm is 6,000 tons.
11. The board of directors of the firm has set four business goals as follows:

    - **Goal 1:** The total production cost is about 32,500,000 DA, with positive tolerance of 1,000,000 DA.
    - **Goal 2:** The total cost of changes in labour levels is about 0 DA, with positive tolerance of 100,000 DA.
    - **Goal 3:** The total carrying cost is about 435,000 DA with positive tolerance of 250,000 DA.

Figure 2 The fluctuation of the actual demand on the level of production capacity for TD, BEN, CAL

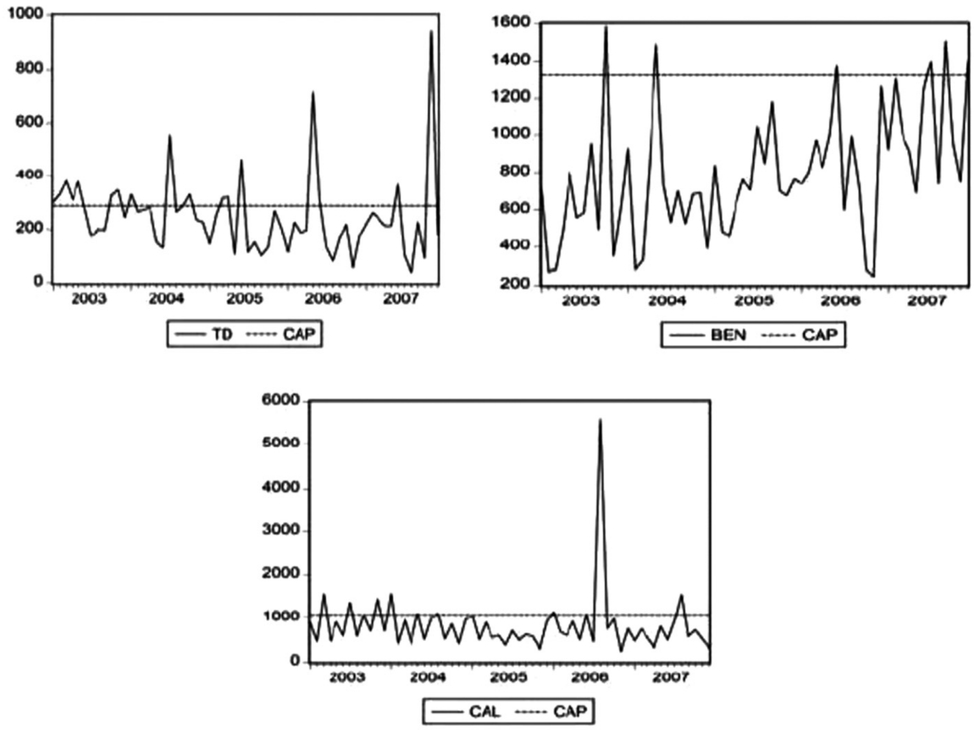


Table 2 The basic data provided by the Bental firm (in units of Algeria dinar DA (1US$≅ 100DA))

product	*Period*	d_{it}	v_{it}	c_{it}	K_{it}
BEN (P_{1t})	1	1377.225	3293.493	208.796	17.794
	2	923.021	3293.493	208.796	15.367
	3	883.342	3293.493	208.796	18.602
	4	1071.99	3293.493	208.796	16.985
	5	1379.269	3293.493	208.796	17.794
	6	1315.222	3293.493	208.796	17.794
TD (P_{2t})	1	128.620	21646.608	848.721	3.883
	2	163.777	21646.608	848.721	3.353
	3	164.617	21646.608	848.721	4.059
	4	166.005	21646.608	848.721	3.706
	5	193.317	21646.608	848.721	3.883
	6	206.662	21646.608	848.721	3.883
CAL (P_{3t})	1	1164.191	1296.109	139.149	14.558
	2	463.447	1296.109	139.149	12.573
	3	659.034	1296.109	139.149	15.220
	4	425.240	1296.109	139.149	13.897
	5	78.967	1296.109	139.149	14.558
	6	478.221	1296.109	139.149	14.558

7.2 Formulation of the RKW-APP

Based on the above information, and using a method (RKW) developed by Kim and Whang (1998) and revised by Yaghoobi and Tamiz (2007-a), the FGP formulation in this study as follows:

$$Min\ \ Z_4 = \frac{1}{3}\beta_1^+ + \frac{1}{3}\beta_2^+ + \frac{1}{3}\beta_3^+$$

Subject to:

$$Z_1 - 1000000\beta_1^+ \le 32000000 \qquad I_{it} \ge 500$$

$$Z_2 - 100000\beta_2^+ \le 0 \qquad \beta_1^+ \le 1$$

$$Z_3 - 250000\beta_3^+ \le 4350000 \qquad \beta_2^+ \le 1$$

$$P_{it} - K_{it} \times W_t \le 0 \qquad \beta_3^+ \le 1$$

$$P_{it} + I_{i,t-1} - I_{it} = d_{it} \qquad I_{10} = 1856.25$$

$$W_t - W_{t-1} - H_t + F_t = 0 \qquad I_{20} = 1029$$

$$W_{Min} \le W_t \le W_{Max} \qquad I_{30} = 1860$$

$$\sum_{i=1}^{3} I_{it} \le 6000 \qquad W_0 = 68$$

$P_{it}, I_{it}, W_t, H_t, F_t, B_1^+, B_2^+, B_3^+ \ge 0$ *i = 1, 2, 3 t = 1, 2.,......, 6*

W_t, H_t, F_t (integers).

7.3 Solving the RKW-APP problem

The LINGO computer software package was used to run the Linear programming model. Table 3 presents the optimal aggregate production plan in the industrial case study based on the current information.

Using the RKW-APP to simultaneously minimise total production costs (Z_1), costs of changes in labour levels (Z_2) and carrying costs (Z_3) yields total production cost of 32,032,504.2 DA, and carrying cost of 4,375,292.99 DA and costs of changes in labour levels of 0. The resulting deviational value for the three fuzzy goal (β_1^+, β_2^+ and β_3^+) are 0.0371, 0 and 0.102 respectively; this means that the membership degrees of the three goals are 0.968, 1 and 0.898, respectively.

Table 3 Optimal production plan in the BENTAL firm case with the RKW-APP model

period	*Product*	P_{it} *(Tons)*	I_{it} *(Tons)*	W_t *(man)*	H_t *(man)*	F_t *(man)*
0	1 (BEN)	–	1865.25	68	–	–
	2 (CAL)	–	1029			
	3 (TD)	–	1860			
1	1 (BEN)	0	679.025	68	0	0
	2 (CAL)	0	900.38			
	3 (TD)	0	695.809			

Table 3 Optimal production plan in the BENTAL firm case with the RKW-APP model (continued)

period	*Product*	P_{it} *(Tons)*	I_{it} *(Tons)*	W_t *(man)*	H_t *(man)*	F_t *(man)*
2	1 (BEN)	743.996	500	68	0	0
	2 (CAL)	0	736.603			
	3 (TD)	267.638	500			
3	1 (BEN)	1074.857	691.515	68	0	0
	2 (CAL)	0	571.986			
	3 (TD)	659.034	500			
4	1 (BEN)	1154.980	774.505	68	0	0
	2 (CAL)	94.019	500			
	3 (TD)	425.24	500			
5	1 (BEN)	1209.992	605.228	68	0	0
	2 (CAL)	193.317	500			
	3 (TD)	78.967	500			
6	1 (BEN)	1209.992	500	68	0	0
	2 (CAL)	206.662	500			
	3 (TD)	478.221	500			

Despite the good results that were obtained through the proposed model, it remains very much sensitive to the accuracy of the information and data provided by the organisation under study.

8 Further scenario designs

This section discusses the actual implementation of the RKW-APP model by considering various alternatives and analysing the sensitivity of decision parameters to variations in relevant conditions, based on the preceding industrial case. The model is implemented in the following seven scenarios.

Scenario 1: Remove Z_3 (carrying costs), consider only Z_1 (total production costs) and Z_2 (costs of changes in labour levels) simultaneously.

Scenario 2: Remove Z_2 (costs of changes in labour levels), consider only Z_1 (total production costs) and Z_3 (carrying costs) simultaneously.

Scenario 3: Remove Z_1 (total production costs), consider only Z_2 (costs of changes in labour levels) and Z_3 (carrying costs) simultaneously.

Scenario 4: Analyse the sensitivity by changing the quantity of tolerance for each goal.

Table 4 shows the implementation data of scenario 4. In Table 4, positive values indicate increases and negative values indicate decreases in related items in each run.
The results of implementing the previous four scenarios are summarised in Table 5 and Table 6. Significant decision making implications for management that were found after sensitivity analysis of the proposed model are as follows:

Table 4 Implementation data of scenario 4

Scenario	*Item*	*Run 1*	*Run 2*	*Run 3*	*Run 4*
Scenario 4	(Tolerance) Δ_{iL}	–30 %	–20 %	+20 %	+30 %

Table 5 Results of implementation in Scenarios 1 to 3

Item	*Scenario 1*	*Scenario 2*	*Scenario 3*
β_1^+	0.06108	0,09881	–
β_2^+	0.04155	–	0.10354
β_3^+	–	0	0.04143
Z_1	32561089,6	32598819,5	–
Z_2	517700	–	103540
Z_3	–	435000	4360358,78

Table 6 Results of implementation in scenario 4

Item	*Run 1*	*Run 2*	*Run 3*	*Run 4*
β_1^+	0,0453	0.0396	0.0264	0.2440
β_2^+	0	0	0	0
β_3^+	0,146	0,128	0.085	0.07881
Z_1	5475055,55	5475055,55	5475055,55	5475055,55
Z_2	4375615,51	4375615,51	4375615,51	4375615,51
Z_3	0	0	0	0

- Comparison of scenarios 1–3 demonstrates the interaction of trade-offs and conflicts among dependent objective functions. From Table 5, it is seen that the total production costs, carrying costs, and costs of changes in labour levels have diverse meanings. For instance, the combination of the total production costs and costs of changes in labour levels in scenario 1 was $Z_1 = 32{,}561{,}089.6$ DA and $Z_2 = 517{,}700$ DA. Moreover, the combination of the total production costs and carrying costs in scenario 2 was $Z_1 = 32{,}598{,}819.5$ and $Z_3 = 435{,}000$ DA. Finally, the combination of the carrying costs and costs of changes in labour levels in scenario 3 was $Z_2 = 103{,}540$ DA and $Z_3 = 4{,}360{,}358.78$ DA. These solutions indicate that a fair difference and interaction exists in the trade-offs and conflicts among dependent objective functions. Different combinations of the arbitrary objective function may influence the objective and β_1^+, β_2^- and β_3^- values. Accordingly, the proposed RKW-APP model meets the requirements of practical application since it can simultaneously minimise the total production costs, carrying costs, and costs of changes in the labour levels.
- The results of scenario 4 indicate that with increase in the quantity of tolerance for each goal, the value for each objective (Z_1, Z_2 , Z_3) remains constant, and its deviational value for the three fuzzy goals (β_1^+, β_2^+ and β_3^+) decreases with decrease in the quantity of tolerance (Δ_{iL}).

9 Conclusions

To conclude our research. we move to present first a brief explanation of APP, which is concerned with determination of production, the inventory and the workforce levels of a company on a finite time horizon. The objective is to reduce the total overall cost to fulfil a situation of inconstant demand, assuming fixed sales and production capacities.

In this study, we used the tolerance approach to the FGP developed by Kim and Whang (1998) and revised by Yaghoobi and Tamiz (2007-a) for aggregate production planning (RKW-APP). The proposed model attempts to minimise total production and work force costs, carrying inventory costs and costs of changes in labour levels, so that in the end, the proposed model is solved by using the LINGO program and getting the optimal production plan.

Moreover, the major limitations of the proposed model concern the assumptions made in determining each of the decision parameters, with reference to production costs, forecast demand, maximum work force levels, and production resources. Hence, the proposed model must be modified to make it better suited to practical applications. Future researchers may also explore the fuzzy properties of decision variables, coefficients and relevant decision parameters in APP decision problems. We will use linear programming with the fuzzy parameters developed by Jiménez et al. (2007) and extended by Marbini.A.H and Tavana M (2011), which will enable us to use the APP problems in cases where the parameters are fuzzy.

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