Nodal solutions to quasilinear elliptic equations on compact Riemannian manifolds

Mohammed Benalili

ABSTRACT. We show the existence of nodal solutions to perturbed quasilinear elliptic equations with critical Sobolev exponent on compact Riemannian manifolds. A nonexistence result is also given.

1. Introduction

In this paper we investigate nodal solutions to quasilinear elliptic equations involving terms with critical growth on compact manifolds. Nodal solutions to scalar curvature type equation has been the subject of investigation by various authors. Among them, we cite D. Holcman[**8**], A. Jourdain [**9**], Z. Djadli and A. Jourdain[**5**]. This work is an extension to a previous one by Z. Djadli and A. Jourdain[**5**] where the authors studied the case of the Laplacian. We use variational methods based on the Mountain Pass Theorem as done in H. Brezis and L. Nirenberg [**3**] and some ideas due to H. Hebey regarding isometry concentration. We approach the problem via subcritical exponents, an idea originated by Yamabe. A non existence result of nodal solution based on a Pohozahev type identity is also given. Let (M, g) be a smooth compact Riemannian manifold $n \geq 3$, with or without boundary ∂M and $p \in (1, n)$. We use the notations of [**5**], let

$$W^{1,p}(M) = \begin{cases} H_1^p(M) & \text{if} \quad \partial M = \phi \\ \overset{o}{H_1^p}(M) & \text{if} \quad \partial M \neq \phi \end{cases}$$

where $H_1^p(M)$ is the completion of $C^\infty(M)$ with respect to the norm

$$\|u\|_{1,p} = \|\nabla u\|_p + \|u\|_p$$

and $\overset{o}{H_1^p}(M)$ is the completion of $C_o^\infty(M)$ with respect to the same norm. Let G be a subgroup of the isometry group of (M, g) denoted $Isom(M)$. We assume that G is compact. We also consider τ an involutive isometry of

2000 *Mathematics Subject Classification.* Primary 58J05.

Key words and phrases. Nodal solutions, quasilnear elliptic equations, mountain pass theorem.