Optimal results for parabolic problems arising in some physical models with critical growth in the gradient respect to a Hardy potential

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Abstract :

We deal with the following parabolic problem

$$\begin{cases} u_t - \Delta u = |\nabla u|^p + \lambda \frac{u}{|x|^2} + f, \quad u > 0 \quad \text{in } \Omega \times (0, T), \\ u(x, t) = 0 \quad \text{on } \partial\Omega \times (0, T), \\ u(x, 0) = u_0(x), \quad x \in \Omega, \end{cases}$$

where $\Omega \subset RN$, $N \geq 3$, is a bounded regular domain such that $0 \in \Omega$ or $\Omega = RN$, $p > 1$, $\lambda \geq 0$ and $f \geq 0$, $u_0 \geq 0$ are in a suitable class of functions.

There are deep differences with respect to the heat equation ($\lambda = 0$). The main features in the paper are the following.

- If $\lambda > 0$, there exists a critical exponent $p_+(\lambda)$ such that for $p \geq p_+(\lambda)$, there is no nontrivial *local*solution.
- $p_+(\lambda)$ is optimal in the sense that, if $p < p_+(\lambda)$ there exists solution for suitable data.
- If we consider the Cauchy problem, i.e., $\Omega \equiv RN$, we find the same phenomenon about the critical power$p_+(\lambda)$ as above. Moreover, there exists a *Fujita type exponent*$F(\lambda) < p_+(\lambda)$ in the sense that independently of the initial datum, for $1 < p < F(\lambda)$, any solution blows up in a finite time respect to an integral norm. This is a major difference with respect to the heat equation ($\lambda = 0$).

Keywords : Quasi-linear heat equations; Existence and nonexistence; Hardy potential; Blow-up; Fujita type exponent.

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