

Elliptic problems with nonlinear terms depending on the gradient and singular on the boundary

Abdellaoui, Boumediene; Giachetti, Daniela; Peral, Irene; Walias, Magdalena

Abstract :

In this paper we consider the problem

$$\begin{cases} -\Delta u = u^{q\alpha} |\nabla u|^q + \lambda f(x) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (\text{P})$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain, $1 < q \leq 2$, $\alpha \in \mathbb{R}$ and $f \geq 0$. We prove that:

- (1) If $q\alpha < -1$, then problem (P) has a distributional solution for all $f \in L^1(\Omega)$, and all $\lambda > 0$.
- (2) If $-1 \leq q\alpha < 0$, then problem (P) has a solution for all $f \in L^s(\Omega)$, where $s > \frac{N}{q}$ if $N \geq 2$, and without any restriction on λ .
- (3) If $q=2$ and $-1 \leq q\alpha < 0$ then problem (P) has infinitely many solutions under suitable hypotheses on f .
- (4) If $0 \leq q\alpha$ and $f \in L^1(\Omega)$ satisfies

$$\lambda_1(f) = \inf_{\phi \in W_0^{1,2}(\Omega)} \frac{\int_{\Omega} |\nabla \phi|^2 dx}{\int_{\Omega} f \phi^2 dx} > 0,$$

then problem (P) has a positive solution if $0 < \lambda < \lambda_1(f)$ and no positive solution for large λ .

Keywords : Elliptic equations; Singular nonlinearities; Dependence on the gradient.

Journal Title / Revue : Nonlinear Analysis: Theory, Methods & Applications, ISSN : 0362-546X, DOI : 10.1016/j.na.2010.10.008, Issue : 4, Volume : 74, pp. 1355–1371, 15 February 2011.