GLOBAL EXISTENCE FOR NONLINEAR PARABOLIC PROBLEMS WITH MEASURE DATA. APPLICATIONS TO NON-UNIQUENESS FOR PARABOLIC PROBLEMS WITH CRITICAL GRADIENT TERMS

B. ABDELLAOUI*, A. DALL'AGLIO, I. PERAL AND S. SEGURA DE LEÓN

Abstract. In the present article we study global existence for a nonlinear parabolic equation having a reaction term and a Radon measure datum:

$$\begin{cases} \big(\varphi(v)\big)_t - \Delta_p\, v &= f(x,t)(1+\varphi(v)) + \mu & \text{in } \ \Omega\times(0,+\infty)\,,\\ v(x,t) &= 0 & \text{on } \partial\Omega\times(0,+\infty)\,,\\ v(x,0) &= v_0(x) & \text{in } \Omega\,, \end{cases}$$

where $1<p<N$, Ω is a bounded open set of $\mathbb{R}^N$ ($N\geq 2$), $\Delta_p u = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ is the so called p-Laplacian operator, $\varphi(s) = \big(1+\frac{s}{p-1}\big)^{p-1}$, $\varphi(v_0)\in L^1(\Omega)$ and μ is a finite Radon measure and $f\in L^\infty(\Omega\times(0,T))$ for every $T>0$. Then we apply this existence result to show wild nonuniqueness for a connected nonlinear parabolic problem having a gradient term with natural growth.

1. Introduction and Statement of Main Results

In this paper we will consider two related problems. The first one is a doubly nonlinear parabolic equation having a reaction term and a measure datum:

$$\begin{cases} \big(\varphi(v)\big)_t - \Delta_p\, v &= f(x,t)\,(1+\varphi(v)) + \mu & \text{in } \ \Omega\times(0,+\infty)\,,\\ v(x,t) &= 0 & \text{on } \partial\Omega\times(0,+\infty)\,,\\ v(x,0) &= v_0(x) & \text{in } \Omega\,, \end{cases} \tag{1}$$

where $f\in L^\infty(\Omega\times(0,T))$ for every $T>0$, $\Delta_p v = \mathrm{div}\,\big(|\nabla v|^{p-2}\nabla v\big)$, with $1<p<N$, μ is a Radon measure whose total variation is finite in $\Omega\times(0,T)$ for every $T>0$, and $\varphi(v_0)\in L^1(\Omega)$; here and in what follows

$$\varphi(s) = \left[\Big(1+\frac{|s|}{p-1}\Big)^{p-1} - 1\right]\ \mathrm{sign}\, s\,. \tag{2}$$

Date: July 11, 2011.

Key words and phrases. Quasilinear parabolic problems, reachable solutions, problems with critical growth in the gradient, measure data and nonuniqueness.

2000 *Mathematics Subject Classification:MSC 2010: 35K15, 35K55, 35B65, 35K65, 35K67.*

The first and the third authors are partially supported by project MTM2010-18128, MICINN, Spain and projet A/030893/10 from A.E.C.I.D., M.A.E. of Spain. The first author is also partially supported by ICTP centre of Italy.

The second and the fourth authors are partially support by the Spanish project MTM2008- 03176.