

## A density-dependent model describing age-structured population dynamics using hawk-dove tactics

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In this paper we deal with a nonlinear two-timescale discrete population model that couples age-structured demography with individual competition for resources. Individuals are divided into juvenile and adult classes, and demography is described by means of a density-dependent Leslie matrix. Adults compete to access resources; every time two adults meet, they choose either being aggressive (hawk) or non-aggressive (dove) to get the best pay-off. Individual encounters occur much more frequently than demographic events, what yields that the model takes the form of a two-timescale system. Approximate aggregation methods allow us to reduce the system while preserving at the same time crucial asymptotic information for the whole population. In this way, we are able to describe the total population size as function of individual aggressiveness level and environmental richness. Model analysis shows a general trend with species that look for richer environment having smaller proportions of hawk individuals with larger costs.

**Keywords:** game dynamics; Leslie model; aggregation methods; individual behaviour; aggressiveness

## 1. Introduction

An important goal of population dynamics is taking into account the behavioural tactics that individuals may adopt to increase their population fitness [11]. In this frame we consider a population structured by age, with juvenile and adult classes, and by behavioural tactics, adults choose between classical hawk and dove tactics in order to get access to a resource.

The principal goal of this paper is analysing the strategies that maximize population fitness. Concerning strategies, we suppose, on the one hand, that adult individuals could adopt an aggressive tactics in order to hoard resources with the drawback of larger costs due to injuries and extra mortality and, on the other hand, they might alternatively use a non-aggressive tactics that entails sharing resources without fight costs. We describe the population dynamics by means of a simple model susceptible of being studied analytically that, at the same time, allows revealing the general trends of our main goal.

The mathematical model takes the form of a discrete system that couples two processes: the adult competition for resources, described by the classical hawk—dove game model [11,14,17,23], and the demography depicted by a density-dependent Leslie-like

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