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## Basic Reproduction Ratio for a Fishery Model in a Patchy Environment

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Abstract We present a dynamical model of a multi-site fishery. The fish stock is located on a discrete set of fish habitats where it is catched by the fishing fleet. We assume that fishes remain on fishing habitats while the fishing vessels can move at a fast time scale to visit the different fishing sites. We use the existence of two time scales to reduce the dimension of the model: we build an aggregated model considering the habitat fish densities and the total fishing effort. We explore a regulation procedure, which imposes an average residence time in patches. Several equilibria exist, a Fishery Free Equilibria (FFEs) as well as a Sustainable Fishery Equilibria (SFEs). We show that the dynamics depends on a threshold which is similar to a basic reproduction ratio for the fishery. When the basic reproduction ratio is less or equal to 1, one of the FFEs is globally asymptotically stable (GAS), otherwise one of the SFEs is GAS.

**Keywords** Population dynamics · Stock-effort model · Time scales · Aggregation of variables · Stability

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