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# SINGULAR ELLIPTIC SYSTEMS INVOLVING CONCAVE TERMS AND CRITICAL CAFFARELLI-KOHN-NIRENBERG EXPONENTS

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Abstract. In this article, we establish the existence of at least four solutions to a singular system with a concave term, a critical Caffarelli-Kohn-Nirenberg exponent, and sign-changing weight functions. Our main tools are the Nehari manifold and the mountain pass theorem.

## 1. Introduction

In this article, we consider the existence of multiple nontrivial nonnegative solutions of the

$$\begin{gathered}
-L_{\mu,a}u = (\alpha+1)|x|^{-2_*b}h|u|^{\alpha-1}u|v|^{\beta+1} + \lambda_1|x|^{-c}f_1|u|^{q-2}u \quad \text{in } \Omega\backslash\{0\} \\
-L_{\mu,a}v = (\beta+1)|x|^{-2_*b}h|u|^{\alpha+1}|v|^{\beta-1}v + \lambda_2|x|^{-c}f_2|v|^{q-2}v \quad \text{in } \Omega\backslash\{0\} \\
u = v = 0 \quad \text{on } \partial\Omega,
\end{gathered} \tag{1.1}$$

where $L_{\mu,a}w := \operatorname{div}(|x|^{-2a}\nabla w) - \mu|x|^{-2(a+1)}w$, $\Omega$ is a bounded regular domain in $\mathbb{R}^N$ $(N \geq 3)$ containing 0 in its interior, $-\infty < a < (N-2)/2$, $a \leq b < a+1$, $1 < q < 2$, $2_* = 2N/(N-2+2(b-a))$ is the critical Caffarelli-Kohn-Nirenberg exponent, $0 < c = q(a+1) + N(1-q/2)$, $-\infty < \mu < \bar{\mu}_a := ((N-2(a+1))/2)^2$, $\alpha$, $\beta$ are positive reals such that $\alpha + \beta = 2_* - 2$, $\lambda_1$, $\lambda_2$ are real parameters, $f_1$, $f_2$ and $h$ are functions defined on $\bar{\Omega}$.

Elliptic systems have been widely studied in recent years, we refer the readers to [1, 7] for regular systems which derive from potential. However, only a few results for singular systems, we can cite [3, 7]. As noticed, when $a = b = c = 0$, $h \equiv 1$, $q = 2$ and $f_1 \equiv f_2 \equiv 1$, Liu and Han [11] studied (1.1). By applying the mountain pass theorem, they proved that, if $0 < \mu \leq \bar{\mu}_0 - 1$ then, system (1.1) admits one positive solution for all $\lambda_1, \lambda_2 \in (0, \eta_1(\mu))$. Here, $\eta_1(\mu)$ denote the first eigenvalue of the positive operator $-\Delta - \mu|x|^{-2}$ with Dirichlet boundary condition. Wu [13] proved that the system (1.1) with $\mu = 0$, has at least two nontrivial nonnegative solutions when $a = b = c = 0$, the pair of the parameters $(\lambda_1, \lambda_2)$ belong to a certain subset of $\mathbb{R}^2$ and under some conditions on the weight functions $f_1$, $f_2$ and $h$. For $= 0$, $= 1$ and $h$ 1, system (1.1) has been studied by Bouchekif and El