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Synthesis of Circular Arrays with Simulated Annealing Optimization Algorithm

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Abstract

This paper presents an analysis and a synthesis of circular arrays of printed antennas. The simulated annealing technique which is a probabilistic methodology to solve combinatorial optimization problems, has been applied to optimize the amplitude of weights coefficients of the elements of the circular array in order to improve the antenna performances and to obtain a beam pattern that meet given requirements. It is seen that the optimization method is effective in designing such arrays. The power of the simulated annealing method lies in its ability to avoid the local minima and to converge to the global minimum of the cost function, thanks to the exploitation of simulated annealing potentialities and high flexibility.

Key words: Printed antennas, circular arrays, analysis, synthesis, simulated annealing.

Introduction

The printed antennas array aroused an interest growing during these last years, in particular in the mobiles communications fields and the monolithic structures, where the radiating elements and the phase-converters are integrated in the same substrate. They also find applications in the space techniques to ensure a specific or partial terrestrial cover, like in the military and civil field. This is mainly due to the unique feature of microstrip antennas; which are, namely, low in profile, compact in structure, light in weight, conformable to non planar surfaces, easy and inexpensive for mass production.

The array association of several printed elements allows in addition an improvement of their performances, to accomplish a very particular functions, such as: scanning and beam steering, jamming rejection, adaptive detection, autoadaptativity, carrying out of various radiation patterns, the directivity pattern and polarization control, …etc.

The circular array, in which the radiating elements are placed on circular rings, is an array of very great practical interest. These applications are multiple: radars, sonar, terrestrial and space navigation and much of other systems [1]-[2].

In the antennas arrays domain, the synthesis problem consists in estimating the variations of the feeding weights of the radiating elements which permits to provide a radiation pattern as close as possible to a desired pattern specified by a shape pattern. The goal of this optimization is to seek for the optimal amplitude according to a precise specification. In this domain, many deterministic synthesis tools were developed. Taking into account the diversity of the goals sake by the users, one will not find a general synthesis method applicable to all cases, but rather a significant number of specific methods to each type of problem. Recently, global stochastic optimization techniques appeared, able to obtain global optima, without remaining trapped on local optima as is in the case for the deterministic methods. To achieve this goal we developed a synthesis method of these arrays by employing an optimization method based on the simulated annealing technique (SA). Simulated annealing is presented as method of finding optimal or near-optimal solutions to problems for which rigorous optimization method do not exist. The literature has reported the application of SA for general electromagnetic problems and, particularly, for the arrays synthesis [3]-[6].

This paper is organized as follows, in section II, the total radiated field of the circular array is determined and plotted in 2D and 3D. Section III presents the synthesis problem. Section IV describes the proposed method of synthesis and its basic concepts are summarized. The obtained results are reported in section V. Finally, conclusions are drawn in section VI.

Analysis

A circular array consists of N radiating elements distributed regularly on a circle of radius R_c, in the *xOy* plane as show in Fig. 1.

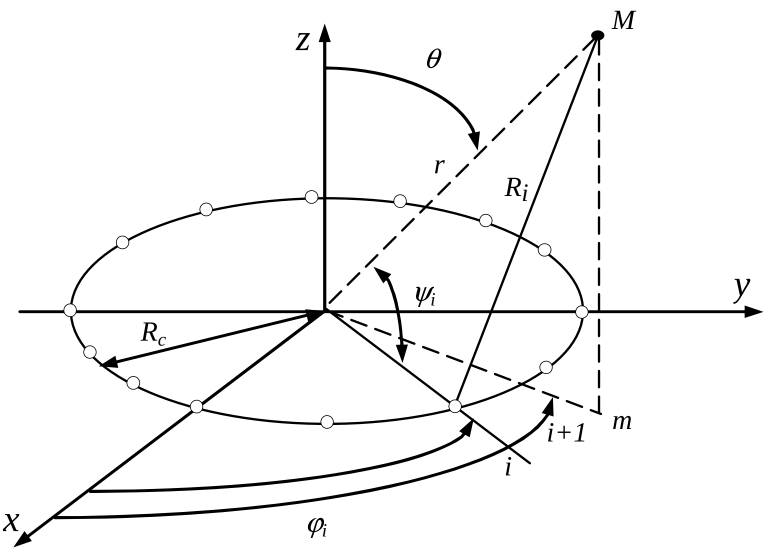


Figure 1: Circular array of N elements.

We consider the total radiated field E_{tot} (r, θ, ϕ) as the result of a sum of the contribution of each element in the observed direction. We can then write [7]:

$$E_{tot}(r,\theta,\phi)=\sum_{i=1}^{N} w_i \frac{e^{-jk_0R_i}}{R_i} E(\theta,\phi) = \frac{e^{-jk_0R_i}}{R_i} \sum_{i=1}^{N} w_i e^{-jk_0R_c\sin\theta\cos(\phi-\varphi_i)} E(\theta,\phi) \quad (1)$$

With:

$\varphi_i = 2\pi\,(\frac{i}{N})$ and $w_i = A_i\, e^{j\alpha_i}$

We can obtain finally:

$$E_{tot}(r,\theta,\phi)=\frac{e^{-jk_0r}}{r}\sum_{i=1}^{N} A_i e^{-j(k_0R_c\sin\theta\cos(\phi-\varphi_i)+\alpha_i)} E(\theta,\phi) \quad (2)$$

To fulfil the electronic sweeping function with such a device and to thus place the main lobe radiation in a direction (θ_0,ϕ_0), it is necessary that the term α_i check the equation (3).

$$\alpha_i = -k_0R_c \sin\theta_0 \cos(\phi_0-\varphi_i) \quad (3)$$

The total radiated field can be written:

$$\begin{aligned} E_{tot}(r,\theta,\phi) &= \frac{e^{-jk_0r}}{r} \sum_{i=1}^{N} A_i e^{jk_0R_c(\sin\theta\cos(\phi-\varphi_i)-\sin\theta_0\cos(\phi_0-\varphi_i))} E(\theta,\phi) \\ &= \frac{e^{-jk_0r}}{r} \times \sum_{i=1}^{N} A_i e^{jk_0d_0\cos(\varphi_i-\zeta)} E(\theta,\phi) \end{aligned} \quad (4)$$

With:

$$\zeta = arctg\left\{\frac{\sin\theta\sin\phi - \sin\theta_0\sin\phi_0}{\sin\theta\cos\phi - \sin\theta_0\cos\phi_0}\right\} \quad (5)$$

and

$$d_0 = R_c\left[(\sin\theta\cos\phi - \sin\theta_0\cos\phi_0)^2 + (\sin\theta\sin\phi - \sin\theta_0\sin\phi_0)^2\right]^{\frac{1}{2}} \quad (6)$$

According to the desired performances, we can consider an array comprising several crowns. Among the various manners of setting out again the elements, one chose a distribution as show in Fig. 2.

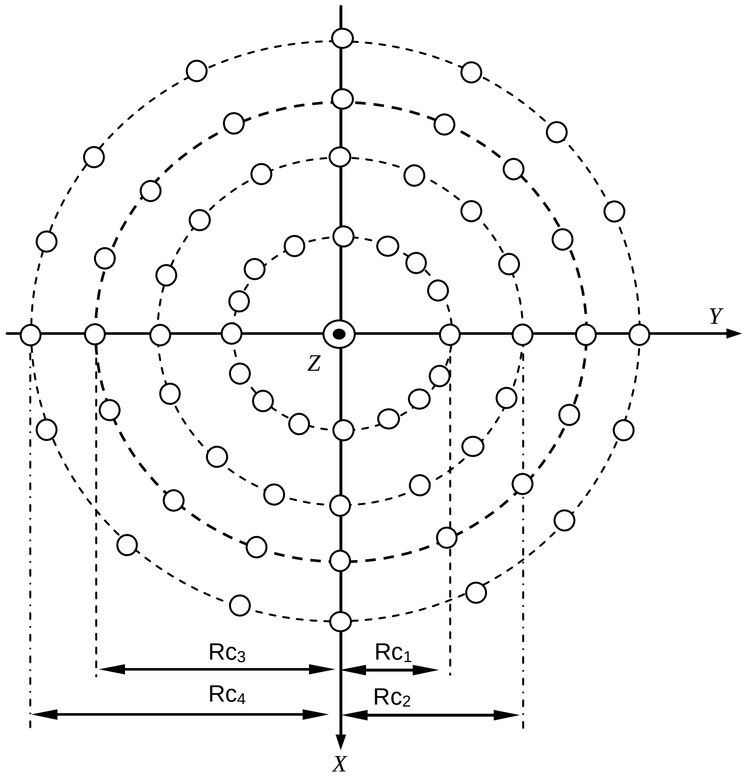


Figure 2: Circular array with several crowns.

The circle which presents the smallest ray must check the weakest spacing between the sources (d_1= 0.6λ). The spacing between the circles is of d_2= 0.6λ (λ being the wavelength number related to the operating frequency).

The total radiated field in this case is:

$$E_{tot}(r,\theta,\phi) = E(\theta,\phi)\sum_{i=1}^{M}\sum_{j=1}^{N} w_{ij} e^{-jk_0 R_{c_i}\sin\theta\cos(\phi-\varphi_{ij})} \quad (7)$$

Synthesis problem

The synthesis problem in which one is interested consists in minimizing the levels of the harmful side lobes to the useful radiation [7]-[10].

$$W_i = A_i e^{-j\varphi_i} \tag{8}$$

Where:

W_i are complex weights to each element.

A_i is the amplitude of the excitation and φ_i is the phase of the excitation.

The synthesis consists of minimizing a criterion of variation δ between the synthesized pattern $F_s(\theta,\varphi)$ and the desired pattern $F_d(\theta)$ defined by a shape pattern imposed in advance by the user. The optimization problem then consists in minimizing the quadratic error.

$$\delta(\theta,\varphi) = \sum_{\theta} \left| F_s(\theta,\varphi) - F_d(\theta) \right|^2 \tag{9}$$

Where :

F_d is the desired function and F_s is the synthesized function.

The SA described in next section, has been found to be very effective for the optimization of the array, thanks to its robustness and inherent ability to accommodate a variety of constraints.

Basic concepts of simulated annealing

SA is a probabilistic method based on concepts deriving from statistical mechanics by the means of the famous method of annealing used by the metallurgists. This method uses the Metropolis algorithm [11]. This algorithm is precisely used to randomly draw a continuation from microscopic configurations by respecting the proportions of Boltzmann relating to balance at a given temperature. As for the algorithm of iterative improvement, the algorithm of Metropolis makes it possible to explore by a random walk a graph whose tops are the microscopic configurations of the system.

In the case of the iterative improvement, displacement in the graph is always carried out towards the configurations of decreasing cost, while the algorithm of Metropolis allows sometimes transitions towards configurations from higher cost. In optimization, an iterative research which accepts only the new points corresponding to a lower value of the function is equivalent to a physical system which reaches temperature equal to zero quickly, which brings us at local minima. On the other hand simulated annealing seeks to converge towards the global minimum thanks to the control of the parameter temperature.

The algorithm of Metropolis calculates the new function $E_{new} = f(x_1)$, with x_1 the new point generated starting from a function $g(\Delta x)$ where Δx is the difference between the new point and the current point.

The majority of the optimization methods using simulated annealing choose their new point with variable distances from their starting point or running. If the solution obtained is better than the preceding one, then this solution is accepted. If the preceding solution remains better, a law of probability of acceptance intervenes in order to decide to keep or reject this value.

Probability of acceptance determined by a function *H*, depends on the temperature *T* and difference between the two values of the function. As an example, while referring to the Boltzmann law, definite as follows [12]:

$$H = \frac{1}{1+\exp(\Delta E / T)} \approx \exp(-\Delta E / T) \tag{10}$$

Where $E = f(x)$ represent the system energy, and $\Delta E = E_{new} - E$ represent the difference in energy between the new point and the preceding point.

In order to accept or to reject a point for which E_{new} is not better than *E*, one carries out the lots of a random variable *P* on [0, 1]. If the variable obtained is lower than *H* the point is then accepted. In the contrary case, the new point is refused.

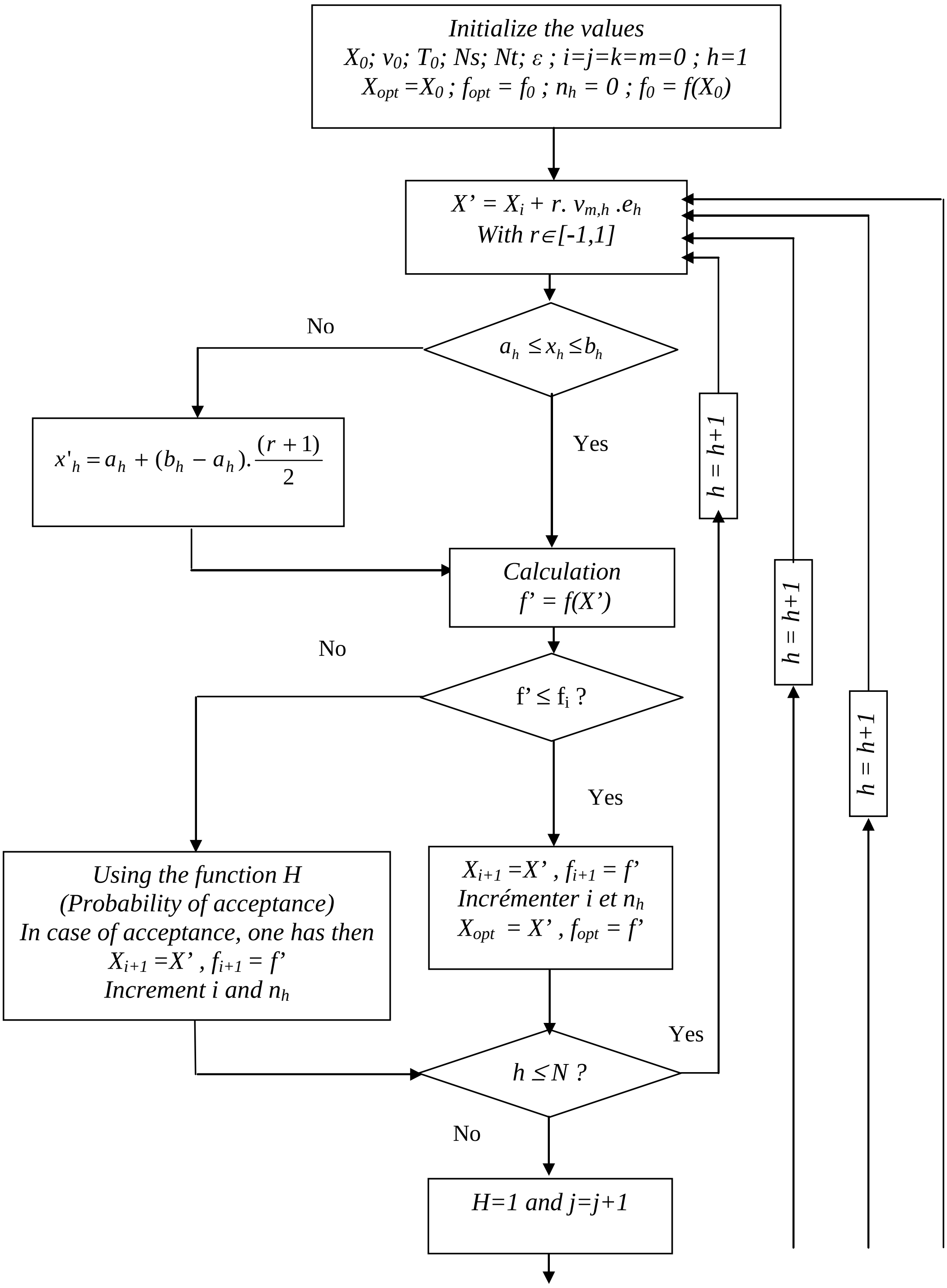
When a new point is accepted, even if the corresponding value of the function is worse than with the preceding point, it becomes then the new point running or solution. At the beginning, the temperature *T* must be large and a new point must be roughly accepted once on two. With the progression of the algorithm in time, the temperature *T* is reduced, implying a fall of the acceptance probability of the points. In fact, the value called ''temperature'' *T* is only one parameter making it possible to control the amplitude of the movements and makes it possible to avoid the minima.

When the temperature is null, the probability of transition becomes unit. If energy decreases at the time of the transformation, and that it is null in the opposite case: the algorithm of Metropolis is then identical to an algorithm of iterative improvement, in this case, one is likely to finish trapped in local minima. On the other hand, when the temperature is not null, the algorithm can choose points with a value of the higher function, which makes it possible to avoid the minima in favour of global minima good located in the workspace.

Simulated annealing algorithms are expected to arrive at a good solution only in a statistical sense as in principale an infinitely large number of iterations are necessary to attain the global minimum. In practice to be useful, an acceptable solution must be attained in a finite reasonable number of iterations. For this to be possible, the cooling schedule must be carefully chosen so that the temperature falls only as fast as is compatible with maintaining a quasi equilibrium, otherwise the algorithm will lock in a secondary minimum.

Various cooling schedules have been experimented with (step by step, linear, geometric and exponential) and, as expected, it is important to cool slowly, particularly at low temperatures. Finally, for the tests described here, the modified exponential cooling schedule recommended by Rees Ball [13] is adopted. However, if a modified exponential scheduling is chosen, almost all process running give slightly different results in term of energy and weight values. This means that the resulting configuration is stable and close to the optimal one.

We used for the synthesis of our radiation pattern the simulated annealing algorithm presented by Corona [14]. This algorithm was tested by various authors and was compared with other techniques like the simplex or gradient conjugate known of the functions comprising local minima. It proved that it always found the global minima which are not the case of the other methods. The algorithm is very simple and is presented in the following general form as shown in Fig. 3.



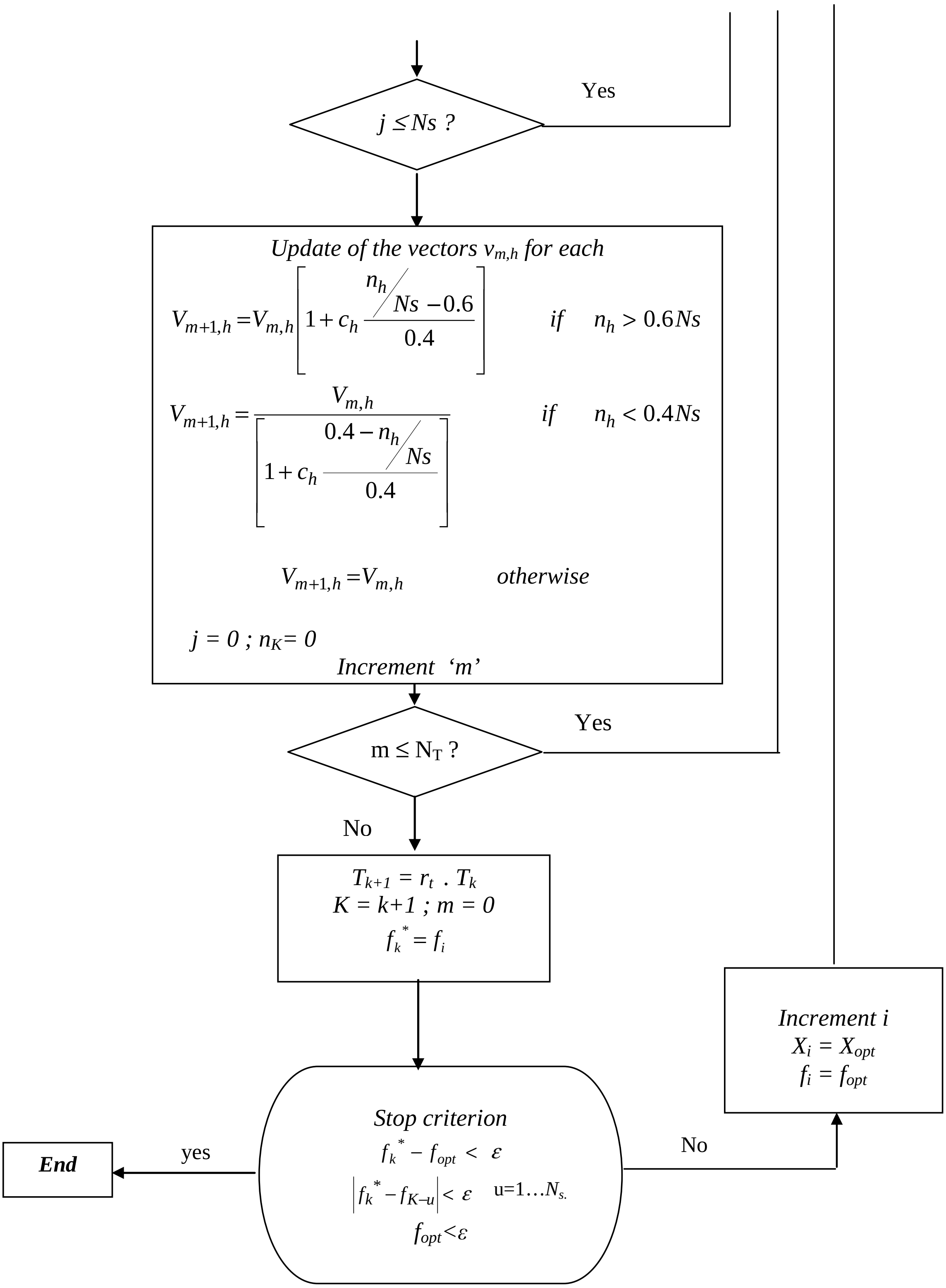


Figure 3: Simulated annealing flow chart.

Synthesis Results

In order to test performances of the proposed approach, we considered circular array with only one crown of 8 elements uniformly spaced at 0.5λ. The shape pattern is specified by an undulation domaine UD_{lim} of -5 dB, for a maximum width of principal beam is of 30° and a minimal width is of 18°, the maximum side lobes level is of -40 dB. The synthesized radiation pattern is shown in Fig. 4.

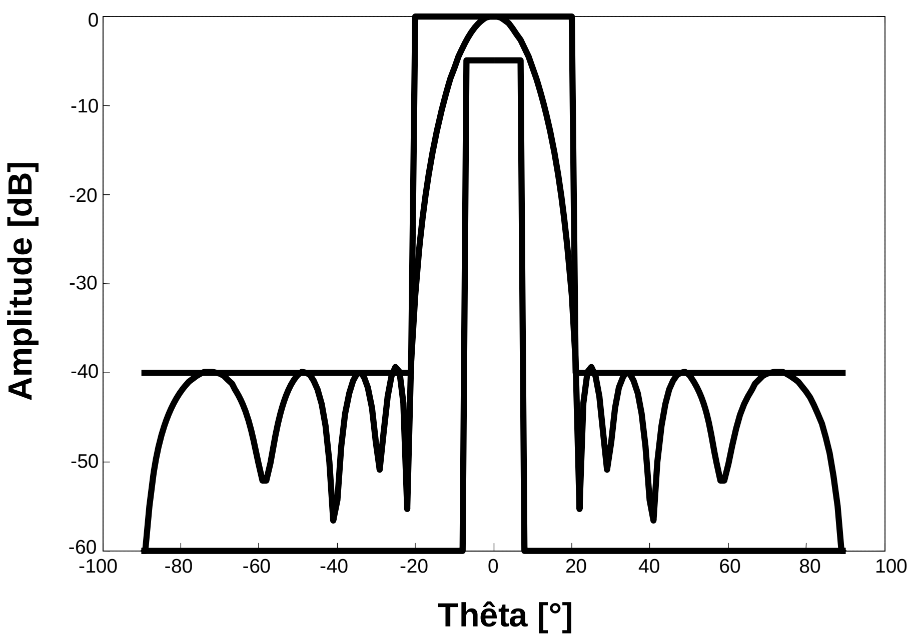


Figure 4: Optimization results of a circular array with one crown (N=8).

Notice that the synthesized radiation pattern is contained within the limits imposed by the desired shape pattern. Most of the power is concentrated around the angle θ = 0º. The main lobe is more directing than that of the linear array and the side lobes maximum is of -40 dB, which remains in agreement with the requirements.

A plot of the error evolution versus iterations is shown in Fig. 5 and Fig. 6 gives the feeding normalized amplitude of the sources.

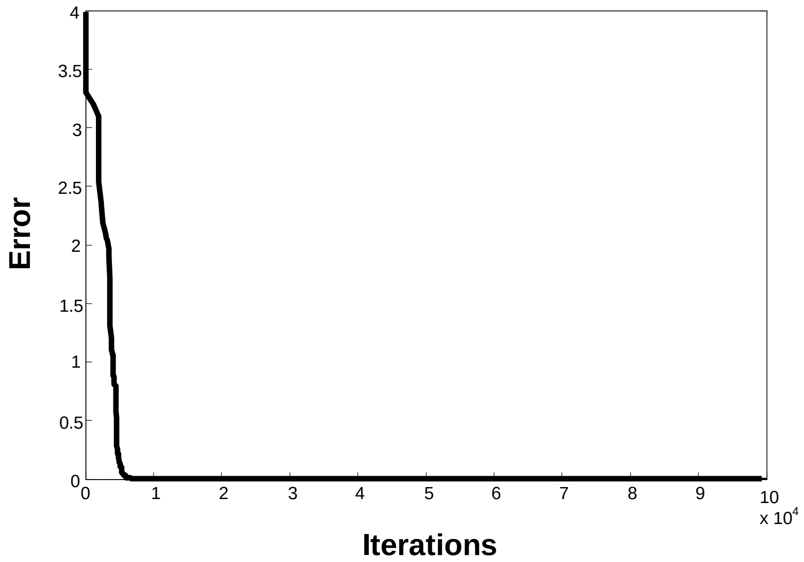


Figure 5: Error versus iterations.

According to the figure above, one notice that the algorithm converges at the end of $11{\times}10^3$ iterations.

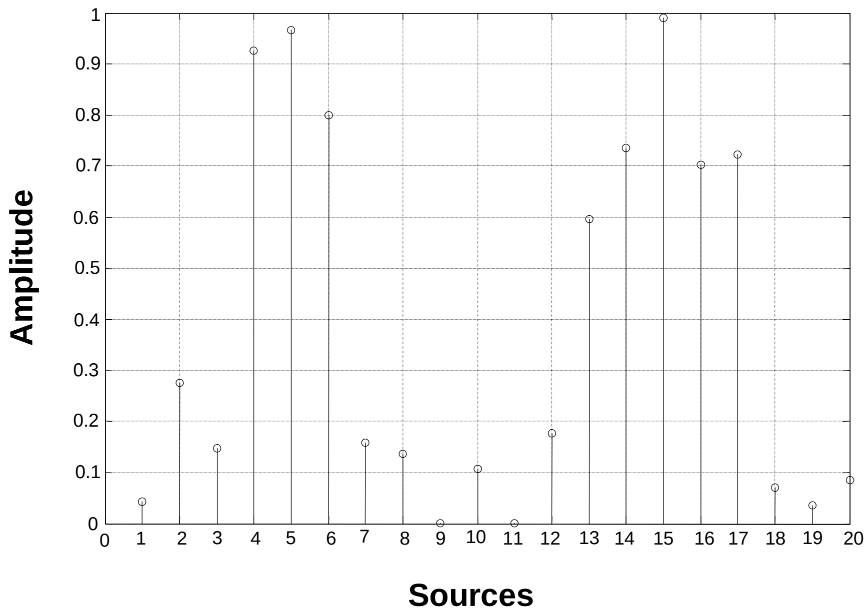


Figure 6: Feeding normalized amplitudes.

On Fig. 7, we show the synthesized radiation pattern of the a circular array with four crowns of 8 (N=32) elements each one.

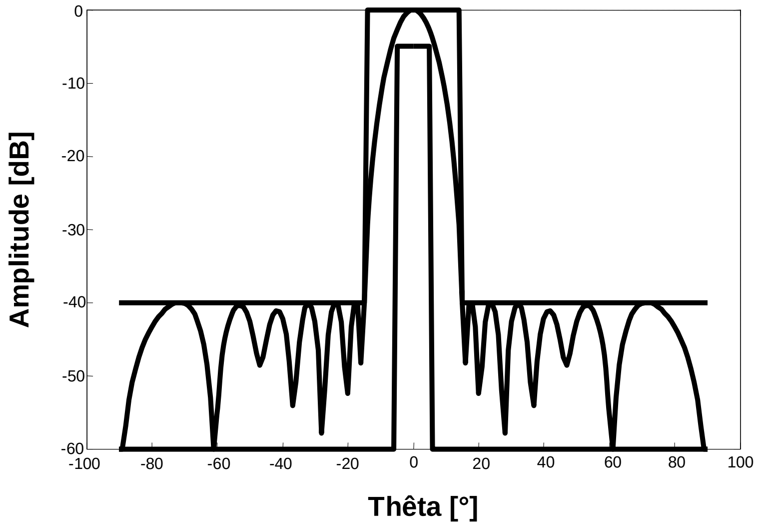


Figure 7: Optimization results of a circular array with four crowns (N=32).

Notice that a perfect symmetrical radiation pattern compared to the reference direction θ = 0º is obtained. The synthesized pattern is contained in the desired shape. Good results was obtained for the side lobes levels where the more dominating is of -40 dB and the maximum tolerable value was fixed at -40 dB.

A plot of the error evolution versus iterations is shown in Fig. 8 and Fig. 9 gives the feeding normalized amplitude of the sources.

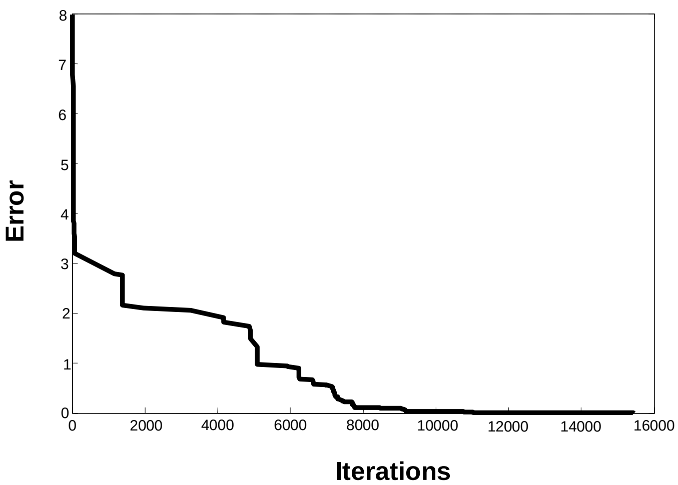


Figure 8: Error versus iterations.

According to the figure above, one notice that the algorithm converges at the end of 11×10^3 iterations.

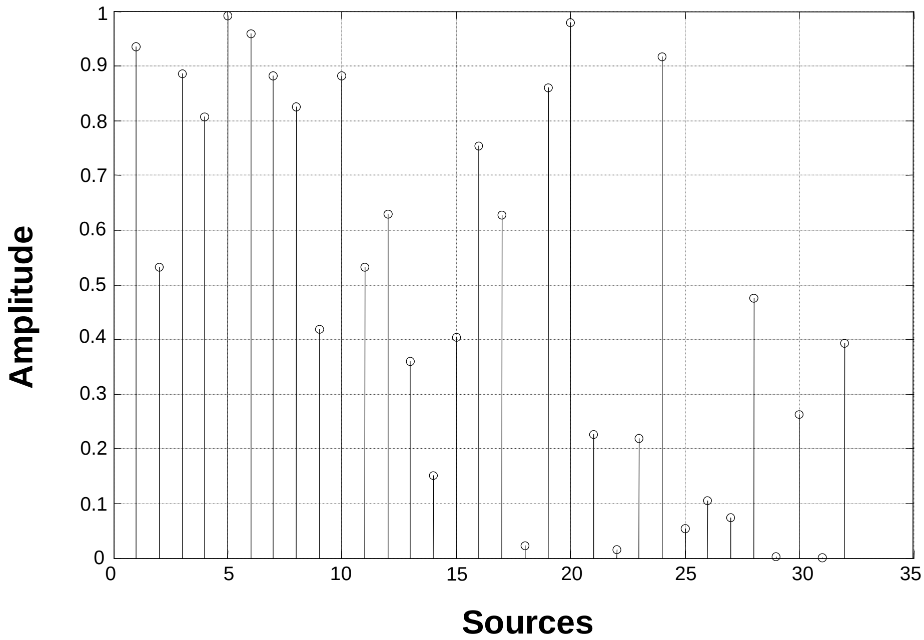


Figure 9: Feeding normalized amplitudes.

Conclusion

In this paper, we developed a global optimization algorithm of circular array of printed antennas based on the simulated annealing method by ordering the amplitude of excitation of each element of the array.

The two cases of arrays that we treated, using a synthesis technique based on the simulated annealing algorithm, substantiate that the application of such an heuristic algorithm achieved the goals of a most rigorous and global approach towards the best solutions. Such solutions remain difficult to achieve using calculus-based on deterministic methods which are too rigid and limited in search space by the local optima difficulties. Moreover, this algorithm is free from all restrictions associated to the integral calculus, derivatives, matrix algebra, discontinuities, etc...

The obtained results are very interesting of share their variety, their general information in the direction of reduction of the side lobes level. This method remains effective and robust towards badly conditioned problems, in particular when the solutions space he comprises discontinuities or constraints on the parameters and especially a great number of local minima. However, the choice of the cost function remains delicate, because the latter represents the key parameter of convergence towards an optimal solution.

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